# Model for cover cracking of RC beams due to partial surface steel corrosion

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## ABSTRACT

Cracking of the cover concrete due to steel corrosion is considered by many researchers to indicate the end-of-service life of corrosion-affected reinforced concrete (RC) structures. Numerous models have been developed to predict the time from corrosion initiation to cracking of the cover concrete. In the previous models, concrete with corroding steel bars was assumed to behave like a thick-walled cylinder under uniform internal pressure. Recent research publications have however, shown that steel corrosion is often concentrated on the surface of the steel that faces the direction of ingress of corrosion agents. This paper presents a model that relates the level of partial surface steel corrosion with the transverse and vertical strains measured on the exterior faces of corrosion-affected RC beams. The model assumes that the remaining section of the steel after corrosion is elliptical shaped. Finally, the model is calibrated with experimental data in the literature and it is shown that assuming uniform steel corrosion underestimates the internal pressure applied by the expansive corrosion products.

## 1. Introduction

First cracking of the cover concrete due to steel corrosion is widely used as an indicator of the end-of-service life of corrosion-affected RC structures. Numerous laboratory experiments have since been and continue to be carried out to assess the time from corrosion initiation to cracking of the cover concrete. Empirical [1], numerical [2] as well as analytical models [3–10] to predict the time for the first appearance of visible corrosion cracks on the cover concrete due to corrosion of the embedded steel have also been developed. The majority of the analytical models are however, too complex to use by structural engineers on the field and yet they often yield poor needed-accuracy.

Previous analytical models on the deformation of the cover concrete due to steel corrosion were primarily based on the principle of a thick-walled cylinder under uniform internal pressure caused by the expansive corrosion products. This assumption might be valid for the accelerated corrosion tests on RC specimens used by the majority of previous researchers where the entire surface of the bar within the desired corrosion region was contaminated with chlorides by either complete immersion of the specimens in NaCl tanks or by casting concrete that was pre-contaminated with chlorides [11–17]. Contrary, in in-service structures, normally one exterior face of the structure is exposed to chloride attack. Works by Malumbela et al. [18,19], Yuan and Ji [20] and Yuan et al. [21] have shown that in such cases, steel corrosion is often concentrated on the surface of the steel bars that faces the direction of ingress of corrosion agents. A term partial surface steel corrosion will be used in this manuscript to differentiate this type of corrosion from the commonly used uniform steel corrosion.

Despite the steel corrosion in [18–20] being concentrated on one surface of corroding steel bars, Malumbela et al. [19] showed that the average rate of steel corrosion under partial surface steel corrosion was comparable to the rate predicted from Faraday's law. Similar results were found by other researchers where uniform steel corrosion was assumed [17,25]. It is logical therefore that for the same theoretical level of steel corrosion (same current density and time of electrolysis), corrosion products and the internal pressure that they apply on the concrete under partial surface steel corrosion will be more concentrated within the corroding section of the bars compared to the corresponding pressure under uniform steel corrosion. In corroboration, works by Malumbela et al. [18] have shown that prior to cracking of the cover concrete, the principal tensile strains due to partial surface steel corrosion were concentrated on the face of the concrete that was exposed to corrosion agents whilst insignificant changes in strains were observed on the faces of beams which were free from ingress of corrosion agents. Clearly, assuming uniform steel corrosion in in-service structures will underestimate the maximum pressure applied by the corrosion process and hence overestimate the time for cracking of the cover concrete.

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## 2. Objectives of the paper

Works by Andrade et al. [2], Oh et al. [13] and Malumbela et al. [18] showed that prior to cracking of the cover concrete, transverse and vertical strains are a better indicator of the progression of the corrosion process compared to the time of first appearance of visible corrosion cracks (which is a once-off measure). This paper therefore attempts to model the transverse and vertical tensile strains applied on the surface of concrete and use them as an easy-to-measure property of corrosion-affected RC structures to indicate the level of steel corrosion. The developed relation will then be calibrated with relevant experimental data in the literature [18,19]. The paper will focus on partial surface corrosion of steel bars and only the period between corrosion initiation and cracking of the cover concrete will be looked at.

## 3. Model description

### 3.1. The porous zone and diffusion of corrosion products

It is accepted that concrete is a porous material and contains voids which corrosion products must first diffuse into before applying stresses on the cover concrete [3–10]. Regrettably, the non-homogeneity of concrete offers a difficult challenge to quantify these voids accurately. As a simplification of the problem (especially when modelling the time to cracking of the cover concrete), the voids in concrete are often represented by a porous zone around the steel bars that has a uniform thickness ranging from 10 μm to 20 μm [3–5,7,9,10].

The assumption of the existence of the porous zone necessitates the relation between the expansion of the cover concrete and the loss in the area of steel during the period from the activation of the corrosion process to the first cracking of the cover concrete to be modelled in two distinct stages. The first stage corresponds to the time required for corrosion products to completely fill the porous zone around the corroding steel bars. During this stage, corrosion products are assumed to diffuse into the porous zone without applying stresses on the cover concrete. It is logical that corrosion products, despite being produced only around the corroding section, will freely flow around the steel so that they completely fill the porous zone.

The second stage is when the porous zone has been fully-filled with corrosion products so that continued steel corrosion necessitates the surrounding concrete to expand so as to allow for deposit of new corrosion products. Unlike in stage 1, it is logical that with the porous zone fully-filled, corrosion products in stage 2 will be accumulated around the corroding area. It is important to note that contrary to partial surface steel corrosion described in this paper, under uniform steel corrosion, corrosion products are assumed to remain uniformly distributed around the surface of the bars even after filling the porous zone.

### 3.2. Corrosion amount to fill the porous zone

Based on works by Malumbela et al. [18] and Yuan and Ji [20], it is reasonable to assume that the remaining section of the corroded steel that faces the direction of ingress of corrosion agents will be elliptical shaped as shown in Fig. 1. If the maximum radius of steel that must be lost to fill the porous zone is, $\Delta r_p$, then the area of steel lost, $\Delta A_{s\_p}$, is given by

$$\Delta A_{s\_p} = \frac{1}{2}\pi r \Delta r_p \tag{1}$$

where $r$ is the radius of uncorroded steel bars.

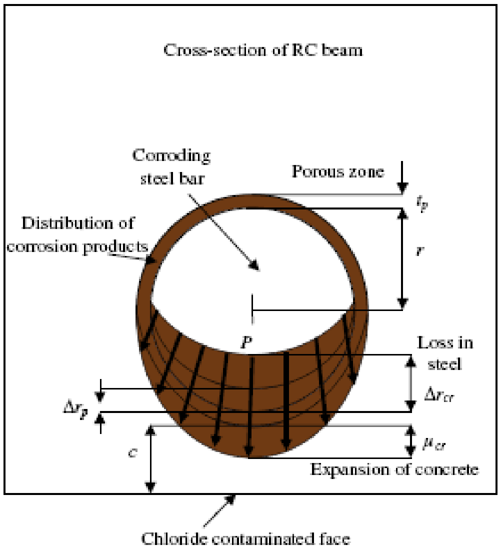

Fig. 1. Partial surface corrosion of steel bars.

Since corrosion products occupy a larger volume that the volume of steel lost, the corresponding volume of corrosion products to fill the porous zone, $A_{cor\_p}$, is given by

$$A_{cor\_p} = \frac{n}{2}\pi r \Delta r_p \tag{2}$$

where $n$ is the ratio of the volume of corrosion products deposited to the volume of steel lost.

Researchers have detected various corrosion products in corrosion-affected RC structures, all with different densities and volume expansion as shown in Fig. 2 [23]. It is well documented that the type of each corrosion product is primarily dependent on the pH and the availability of oxygen [10,23–25]. These factors (pH and quantity of oxygen) are extremely variable and difficult to quantify in a corrosion-affected RC structure. According to various researchers, for corrosion of steel that is embedded in concrete, ferrous hydroxide is the fundamental corrosion product [10,23–25]. However, with an increase in the supply of oxygen (especially after cracking of the cover concrete), more stable corrosion products such as haematite and magnetite are formed. Interestingly, the volume expansion of ferrous hydroxide is about 1.7 times larger than the volume expansion of the more stable compounds [10,23,24].

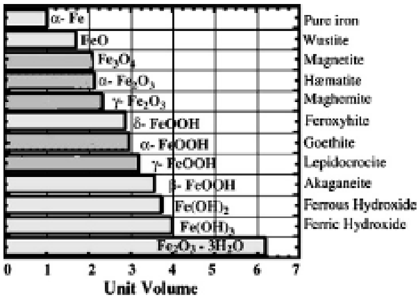

Fig. 2. Corrosion products of iron.

When modelling the time to cover cracking of concrete due to steel corrosion, researchers have therefore found it convenient and conservative to use ferrous hydroxide as the primary corrosion product [3–5,7–10]. Moreover, accelerated corrosion tests in laboratory RC specimens often involve full immersion of the specimens [11–14] or cyclic wetting of the specimens [18,19] with salt water. It is logical therefore that prior to cracking of the cover concrete, the supply of oxygen to the corrosion region will be compromised so that the dominant corrosion product at that stage will be ferrous hydroxide. In corroboration, for the RC beams with the test results that will be used in this paper, the corrosion products observed around the steel when the beams were opened for repair were greenish-black in colour indicating a large presence of ferrous hydroxide. In the contrary, reddish-brown products were found after testing the beams to failure indicating a large presence of the more stable corrosion products such as haematite.

As previously mentioned, during the process of filling the porous zone, corrosion products are expected to freely flow around the steel bars till the porous zone is fully-filled with the products. Assuming that the porous zone has a uniform thickness, $t_p$, around the corroding steel as in [3,4,7,10], it can easily be shown from Eqs. (1) and (2) and the resultant volume of the porous zone that to fill the zone with corrosion products, the maximum radius of steel lost, $\Delta r_p$, is given by

$$\Delta r_p = \frac{2t_p(2r + t_p)}{r(n-1)} \tag{3}$$

The corresponding level of steel corrosion (as a percentage mass loss of steel) necessary for corrosion products to apply stresses on the internal surfaces of the concrete can be calculated from Eqs. (1)–(3). In addition, knowing the rate of steel corrosion, the time required to fill the porous zone (which some researchers such as [7,10] refer to as the free expansion period) can be determined.

### 3.3. Expansion of the cover concrete at the time of cracking

As discussed above, when the porous zone is fully-filled, additional corrosion products apply tensile stresses on the surrounding concrete and eventually cause cracking of the cover concrete. During this stage, the corrosion products are expected to accumulate only around the corroding section of the steel and most importantly, to mirror the elliptical shape of loss in the section of steel as shown in Fig. 1. Conversely, the expansion of concrete is also expected to copy the shape of the accumulation of the corrosion products. As previously discussed, this assumption is validated by the principal transverse strains recorded on the surface of the concrete that faces the direction of ingress of corrosion agents [18].

Let the maximum radial expansion of the concrete that surrounds the corroding steel bars at the time of cracking of the cover concrete be $\mu_{cr}$ and the corresponding maximum loss in the area of steel be $\Delta r_{cr}$. Similar to Eq. (1), the area of steel lost at the time of cracking of the cover concrete is given by Eq. (4) whilst the corresponding volume of corrosion products deposited ($A_{cor\_cr}$) is given by

$$\Delta A_{st\_cr} = \frac{1}{2}\pi r \Delta r_{cr} \tag{4}$$

$$A_{cor\_cr} = \frac{n}{2}\pi r \Delta r_{cr} \tag{5}$$

From Eqs. (4) and (5) and from Fig. 1, it can be shown that the maximum expansion of concrete near the corroding area, $\mu_{cr}$, necessary to accommodate the corrosion products, is given by

$$\mu_{cr} = \frac{(n-1)r\Delta r_{cr} - 2t_p(2r + t_p)}{r + t_p} \tag{6}$$

It should be noted that Eq. (6) is valid only when the porous zone is fully-filled or if presented mathematically, when $(n-1)r\Delta r \geqslant 2t_p(2r + t_p)$.

From previous works, it is reasonable to assume that at the early corrosion stages, the rate of expansion of concrete is linearly related to the rate of steel corrosion. Moreover, studies by Andrade et al. [2], Oh et al. [13], and Malumbela et al. [18] where the rate of steel corrosion was controlled by impressing a constant direct current, showed that at the early corrosion stages (before cover cracking), there is a linear time-variation of strains measured on the exterior surface of the concrete. It is also reasonable to assume that the cover concrete behaves like a thick-walled cylinder with varying internal pressure around the surface of the corroding bars that is dependent on the level of steel corrosion. Under this assumption, internal pressure is expected to be maximal on the surface of the corroding steel bars that faces the direction of ingress of corrosion agents. Conversely, little pressure is expected on the surfaces of the steel bars that are opposite the direction of ingress of corrosion agents (Fig. 1). From basic mechanics and Fig. 1, the theoretical maximum radial expansion of concrete near the corrosion products is therefore given by [26]

$$\mu_{cr} = \frac{P(r + t_p)}{E}\left[\frac{(r + t_p)^2 + (r + c + t_p)^2}{(r + c + t_p)^2 - (r + t_p)^2} + \nu\right] \tag{7}$$

where $P$ is the internal pressure applied on the inner surfaces of the cylinder by the expansive corrosion products; $E$ is the modulus of elasticity of concrete; and $\nu$ is the Poisson's ratio of concrete.The corresponding theoretical maximum transverse strain on the surface of the concrete, $\varepsilon_{ext}$ is given by [26]

$$\varepsilon_{ext} = \frac{2P(r + t_p)^2}{E[(r + c + t_p)^2 - (r + t_p)^2]} \tag{8}$$

Having developed the model, the following is a procedure for calculating the loss in the area of steel when knowing the external applied strains (which can easily be measured on the surface of the concrete).

1. Use Eq. (8) to calculate the ratio between the maximum internal pressure, $P$, applied by the corrosion products and the modulus of elasticity of concrete, $E$.
2. Use Eq. (7) to calculate the maximum radial expansion of the concrete near the corroding area, $\mu_{cr}$.
3. The maximum loss in the radius of steel, $\Delta r_{cr}$, can then be calculated from Eq. (6).
4. Finally, the loss in the area of steel can be calculated from Eq. (4).

It is important to note that the model was developed for uncracked concrete which according to Malumbela et al. [18], corresponds to external strains that are less than 300 micro strains. The deformation of the cover concrete after cracking is fundamentally different as discussed in [18].

## 4. Calibration of the model

To calibrate the presented model that relates the transverse and vertical strains on the exterior surfaces of concrete with the amount of steel corrosion prior to cracking of the cover concrete, the previous test results for the rates of transverse and vertical strains and the rate of loss of steel in Figs. 3 and 4 were used [18,19]. Since the rate of steel corrosion is dependent on the availability of corrosion agents, the lowest rate of steel corrosion is expected prior to cracking of the cover concrete when oxygen around the corroding area is scarce. From Fig. 4, the lower average mass

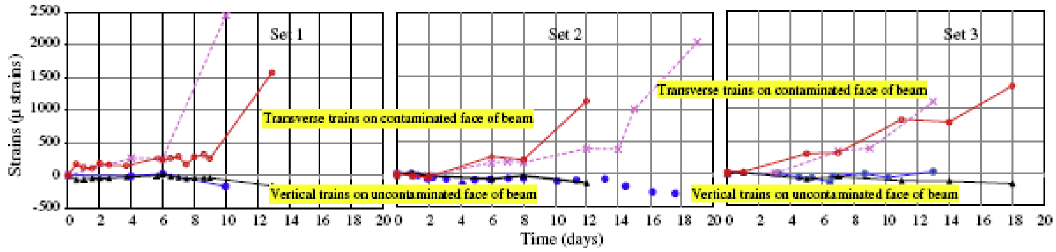

**Fig. 3.** Transverse and vertical strains before cracking of cover concrete.

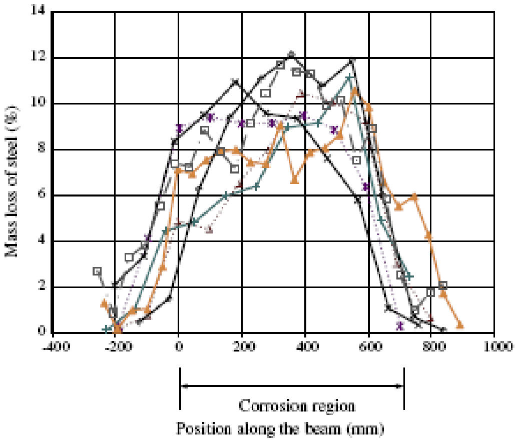

**Fig. 4.** Variation of mass loss of steel bars (after 65 days) along beams used to calibrate the model.

loss of steel for the various beams was 8%. As outlined in [19,22], the duration of the accelerated corrosion test was 65 days. Assuming mass loss of steel linearly increased over the corrosion test because the current density induced was constant, the rate of steel corrosion was 0.12% per day.

It was discussed in [18] that visible cracks of the cover concrete were observed after about 7 days of corrosion testing. It was however, argued in the same paper that first cracking occurred prior to the appearance of visible corrosion cracks (after 4 days) when the principal transverse strains on the surface of the cover concrete were about 300 micro strains (Fig. 3). At a corrosion rate of 0.12% per day, cracking of the cover concrete therefore occurred at a mass loss of steel of 0.49%.

If the above discussed procedure for calculating the loss in the area of steel is used with the assumptions that; the average thickness of the porous zone is 15 µm as used in [3,4,7,10]; the ratio of volume of corrosion products to the volume of steel lost is 3.21 as used in [3,4,7,10]; the Poisson's ratio of concrete is 0.18 as used in [3–10]; and cracking occurs when the strains on the exterior surface of concrete is 300 micro strains as found in [18] then the theoretical mass loss of steel after 4 days is 0.46% which is close to the measured mass loss of steel. If however, uniform steel corrosion was assumed instead of partial surface steel corrosion, it can be shown that to obtain the external strains of 300 micro strains for beams that were used in [18,19,22], the level of steel corrosion must be 1.2%. Since to obtain a given strain on the external surface of a corrosion-affected RC structure, uniform steel corrosion requires a larger mass loss (about three times more) compared to partial surface steel corrosion, assuming uniform steel corrosion underestimates the maximum internal pressure applied by corrosion products under partial surface steel corrosion.

## 5. Conclusions

A model that relates the level of steel corrosion (which is difficult to measure) with the transverse strains applied on the exterior faces of corrosion-affected RC beams (which is easy-to-measure) due to partial surface steel corrosion was presented in the paper. Contrary to previous models that assumed uniform steel corrosion, the model assumed that; steel corrosion is concentrated on the surface of steel that faces the direction of ingress of corrosion agents; after filling the porous zone between corroding steel and the surrounding concrete, corrosion products and the internal pressure are concentrated on the corroded surface of the steel; and the remaining section of the steel, the variation of accumulated corrosion products, and the variation of the internal pressure are elliptical shaped. The model was shown to better relate the strains previously recorded on the surface of corroded RC beams in the literature with the level of steel corrosion compared to assuming uniform steel corrosion. Due to lack of data in the literature, this model was calibrated only with previous results from the current authors [18,19]. It is important that with more data in the future, the model be re-calibrated especially to model cover cracking under natural steel corrosion.

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