Total Solar Radiation Received at Various Collector Inclinations

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ABSTRACT

Solar energy is finding wide application via several energy-saving devices and systems. Its use, especially in rural areas, can often be limited by the non-availability of complete information on insolation data. In this investigation, both the mean extraterrestrial and terrestrial radiations were calculated for three zones in Nigeria. These zones were assumed to be dependent mainly on the latitude, L. *The isotropic model was employed for the computation of the diffuse and ground reflected components of radiation. Consequently, the total insolation was obtained for various inclinations of the collector surface. It was found that three solar energy seasons exist for each zone. For maximum total incident radiation, a different collector inclination is ideal for each solar energy season. However, for all year round applications, a fixed inclination varying between* L *and (*L+*10) degrees is recommended.*

NOTATION

a, b	Constants in the Ångström relationship
E	Orbital eccentricity factor
H	Monthly radiation, MJ/m^2 day
I_{sc}	Solar constant, kW/m^2
Kt	Clearness index $(=H_2/H_1)$
L	Latitude (degrees)
n	Number of days beginning from January 1 as unity
R	Collector's tilt factor
R_b	Tilt factor for beam radiation
R_g	Tilt factor for ground reflected radiation
R_s	Sky-view correction factor

S' Non-dimensional sunshine hours
t_{sr} Minimum between local or apparent sunrise hour angle(degrees)

β Collector inclination (degrees)
δ Solar declination (degrees)
ρ Ground albedo
Ω Local sunrise hour angle (degrees)
Ω' Apparent sunrise hour angle (degrees)

Subscripts

1 Extraterrestrial
2 Terrestrial
b Beam value on horizontal surface
bt Beam value on tilted surface
d Diffuse
g Ground reflected
s Sky reflected
t Total value on an inclined surface
ta Annual total value

1 INTRODUCTION

The use of solar energy continues to be attractive for various applications. In many countries, one major constraint to the utilization of solar energy is the non-availability of composite radiation data. These data are required for the proper design and sizing of solar-equipment components such as for water heaters, photovoltaic systems, communication devices, dryers, stills and pumping devices.

In Nigeria, measurements and the prediction of insolation data for horizontal surfaces have been carried out for some locations. Ezekwe and Ezeilo[1] were probably the earliest researchers to propose an empirical equation to predict the solar radiation for two cities in Nigeria. Bamiro[2] used some empirical equations relating solar radiation to climatological variables to obtain the mean insolation for Ibadan. He observed that the tested models produced results which were in good agreement with measured data. He then proposed a new single model for predicting monthly total radiation.

Ideriah and Suleiman[3] determined the clearness index under various atmospheric conditions in Ibadan. They used the results to show that there are six different patterns for the annual sky conditions. Oladiran[4]

TABLE 1
Locations of Zones and Stations

Zone	*Latitude of zone* (°)	*Sample station*	*Latitude of station* (°)
1	4–7·5	Lagos	6·33
2	7·5–10	Ilorin	8·48
3	10–14	Maiduguri	11·85

analysed measurements from 16 Nigerian agrometeorological stations using the Ångstrom model. Because of the significant effect of latitude on radiation in Nigeria, he proposed that the country be zoned into three and correlation equations were obtained for these zones. Kuye and Jagtap[5] analysed the global radiation measured at Port Harcourt and identified five seasonal variations from the clearness-index data.

Data on measured radiation for inclined planes are often scanty. Yet these data are often required for the architectural design of buildings (e.g. in the optimization of roofs and orientation of walls) and in the design and evaluation of tilted flat-plate solar collectors. However, some researchers have calculated the magnitude of solar radiation on tilted surfaces for various countries.[6–9]

In the present study, the rate of insolation on inclined surfaces was obtained from theoretical models. First, a characteristic location was chosen for each of the three zones identified by Oladiran.[4] These are Maiduguri, Ilorin and Lagos for the northern, the middle and the southern zones of Nigeria, respectively—see Table 1. The total radiations falling on tilted surfaces in each zone were calculated and were assumed to be representative of all sites in the zone.

2 THEORETICAL BACKGROUND

The extraterrestrial radiation on a horizontal plane can be computed from

$$H_1 = 7.64 I_{sc} E \sin\beta \sin\delta [\Omega - \tan\Omega] \quad (1)$$

where the local sunrise hour angle, Ω is

$$\Omega = \cos^{-1}(\tan L \tan\delta) \quad (2)$$

and

$$\delta = 23.45 \sin[360(284+n)/365]$$

The apparent sunrise hour angle for a tilted surface, which faces due south, is given by

$$\Omega' = \cos^{-1}[\tan(L-\beta)\tan\delta] \quad (3)$$

The monthly averaged solar flux on a horizontal surface was obtained by using the Ångstrom-type equation:

$$H_2=H_1[a+bS'] \tag{4}$$

This global horizontal radiation consists of both the diffuse, H_d, and beam or direct, H_b, components, which can be determined from

$$H_2=H_d+H_b \tag{5}$$

There are several correlation relations for calculating the diffuse component. The accuracy of various relations for the Nigerian weather conditions is discussed in Oladiran.[10] However, a mean of the correlations proposed by Collares-Pereira and Rabl,[11] Liu and Jordan[12] and Page[13] was employed in this study.

The total long-term solar flux, H_t, intercepted by a tilted plane is the sum of the beam radiation, H_{bt}, the diffuse sky component, H_s, and the ground reflected radiation, H_g. Mathematically,

$$H_t=H_{bt}+H_s+H_g \tag{6}$$

The beam radiation incident on an inclined surface is directly proportional to the beam radiation on the plane when horizontal and is given by

$$H_{bt}=R_bH_b \tag{7}$$

where

$$R_b=\frac{\cos(L-\beta)\cos\delta\sin t_{sr}+t_{sr}\sin(L-\beta)\sin\delta}{\cos L\cos\delta\sin\Omega+\Omega\sin L\sin\delta} \tag{8}$$

The value of t_{sr} in eqn (8) is defined as the time when the solar collector starts to receive direct thermal energy and is the smaller of either the local or apparent sunrise hour angles, i.e.

$$t_{sr}=\min[|\Omega'|,|\Omega|]$$

The sky-diffuse and the ground-reflected radiation components can, respectively, be expressed as

$$H_s=R_sH_d \tag{9a}$$

and

$$H_g=\rho R_gH_2 \tag{9b}$$

where R_s is the sky-view correction factor. Assuming the sky-diffuse solar radiation to be isotropic, we have

$$R_s=(1+\cos\beta)/2$$

The tilt factor for ground-reflected radiation is

$$R_g=(1-\cos\beta)/2$$

and ρ, the ground reflectance (albedo) is the ratio of ground-reflected solar radiation and the global radiation incident on the ground. A constant ground albedo of 0.26 for green grass surfaces was assumed for various seasons in all the zones in this investigation. Any variation due to non-availability of either the measured albedo or actual spectral hemispheric reflectance was assumed to be minimal for the present purposes.

The collector tilt factor, *R*, was obtained from a simple relationship:

$$R=H_t/H_2 \quad (10)$$

Substituting eqns (6), (7) and (9) into eqn (10), we have

$$R=(H_b/H_2)R_b+(H_d/H_2)R_s+\rho R_g \quad (11)$$

Computer programs were written to solve eqns (1) to (11) while the collector inclination was varied from (*L*–20) to (*L*+30) degrees. The horizontal configuration corresponds to an inclination of zero.

3 RESULTS AND DISCUSSION

Employing eqns (1) and (4), the average monthly extraterrestrial radiation and global radiation on a horizontal surface were calculated. The total monthly and average annual radiations incident on non-horizontal surfaces were then computed. The maximum sunshine was obtained by integrating eqn (2).

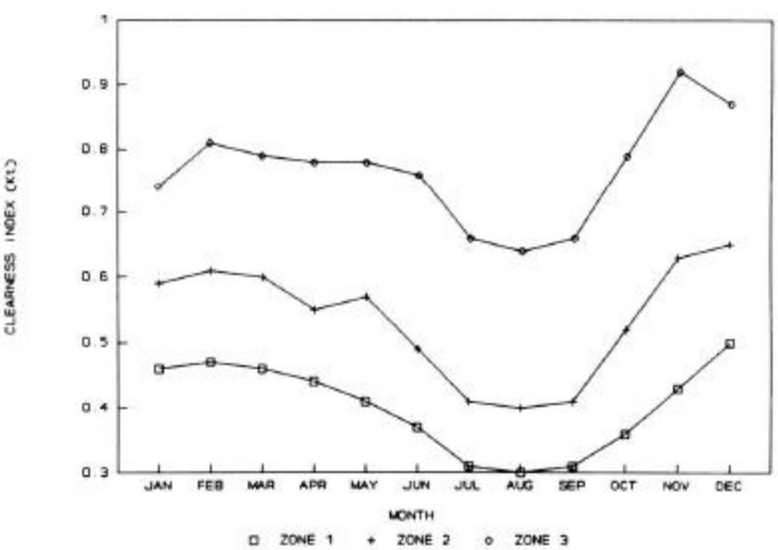

Fig. 1. Variation of clearness index.

The monthly average of daily cloudiness or clearness index (H_2/H_1) is depicted in Fig. 1. Generally, the clearness index decreases from February, reaching the minimum value between July and September and then increases until the maximum value is attained between November and December. This maximum varies from about 0.92 in zone 3 to about 0.50 in zone 1. Similarly, the minimum value varies from 0.3 to 0.65 from zone 1 to zone 3. There are two main seasons in Nigeria, namely the rainy season and the dry season. These occur, respectively, between (i) March and September and (ii) October and February. The rainy season is usually characterized by heavy suspended water vapour in the atmosphere and severe cloudiness. The dry season is associated with high temperatures and clearness of the sky. Thus, the occurrence of the maximum and minimum values of clearness index during the dry and rainy seasons, respectively, is in agreement with climatological observations.

It can be seen that the monthly clearness index is very low for sites in zone 1 so that the bulk of the extraterrestrial radiation is attenuated. This may be due to either a large amount of absorption by atmospheric gases or scattering by air molecules or aerosols. The corresponding values for zone 3 are usually high so that the temperatures in this zone are high.

Figure 2 shows the variation of the theoretical maximum sunshine hours. The peak values are 12.7, 12.5 and 12.3 h for zones 3, 2 and 1, respectively. However, the actual sunshine hours are much less, especially during the months of March through September.

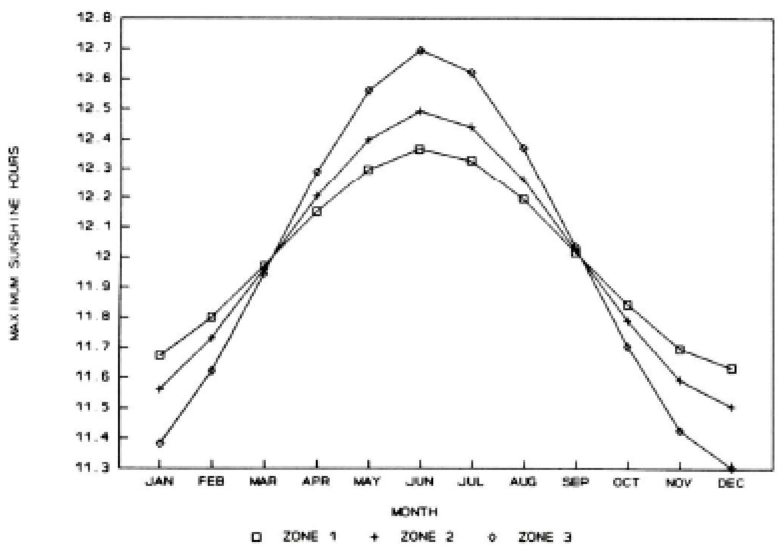

Fig. 2. Variation of monthly maximum sunshine hours.

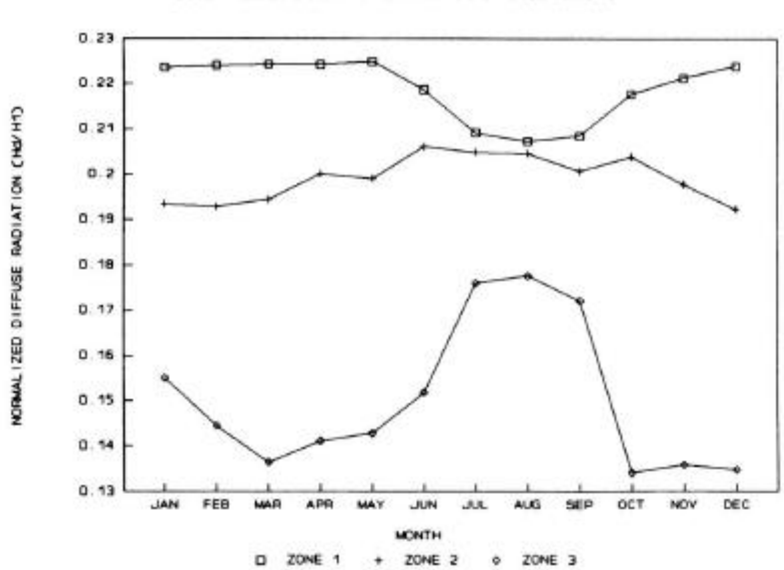

Fig. 3. Variation of diffuse radiation.

The monthly ratio of diffuse radiation to extraterrestrial radiation is presented in Fig 3. The values for zones 1 and 2 seem to be almost invariant at 0.22 and 0.20, respectively. Ideriah and Suleiman[3] observed that this factor for Ibadan was fairly constant at 0.24. Ibadan is a city located in zone 1. Further comparisons could not be made as there are no known measured values of diffuse radiation for locations in this report.

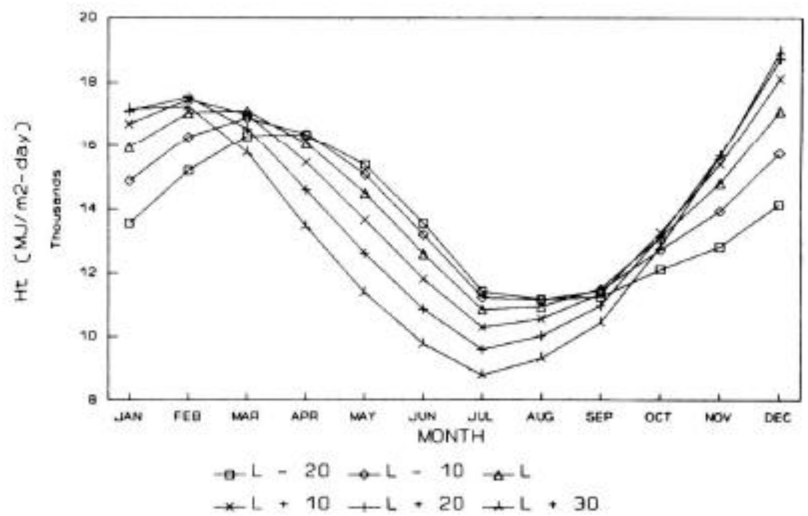

Fig. 4. Monthly distribution of terrestrial radiation for zone 1.

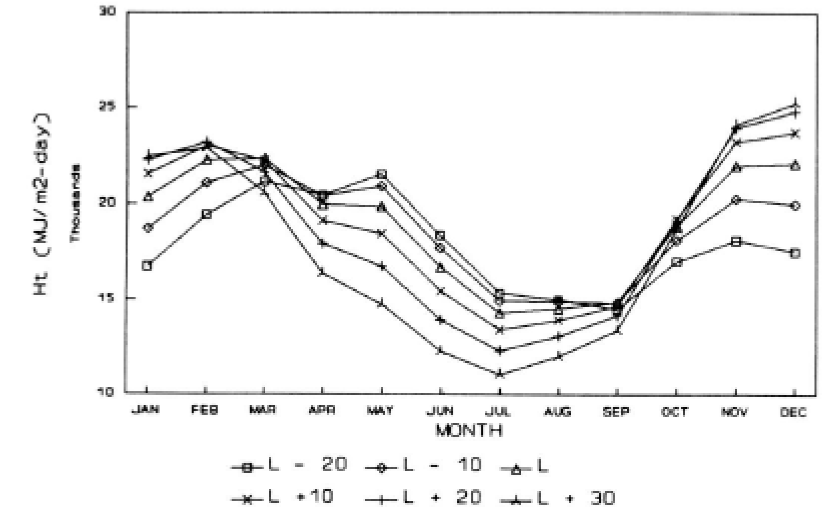

Fig. 5. Monthly distribution of terrestrial radiation for zone 2.

Variations of the mean monthly total radiation incident on tilted surfaces are presented in Figs 4 to 6. It can be observed that the pattern of the figures is fairly similar for all the zones. The primary and secondary peaks occur between November and December, and between February and May, respectively. The minimum insolation generally occurs in July. Although there are two main characteristic weather seasons in Nigeria,

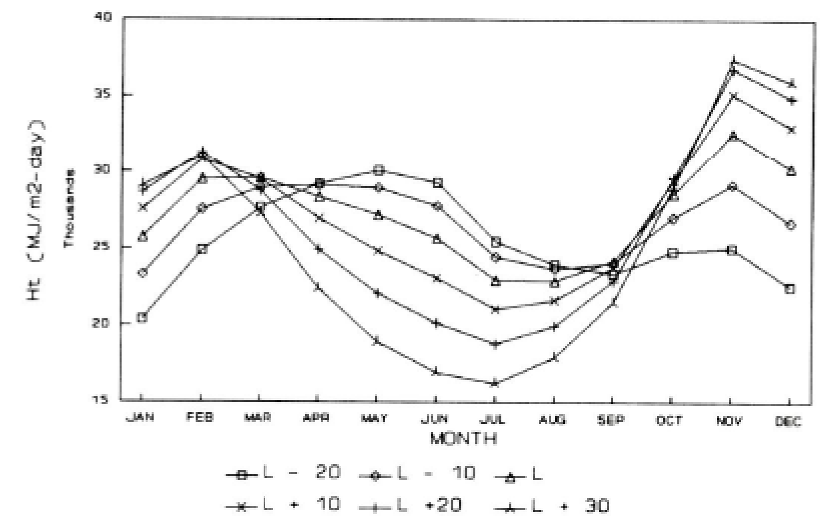

Fig. 6. Monthly distribution of terrestrial radiation for zone 3.

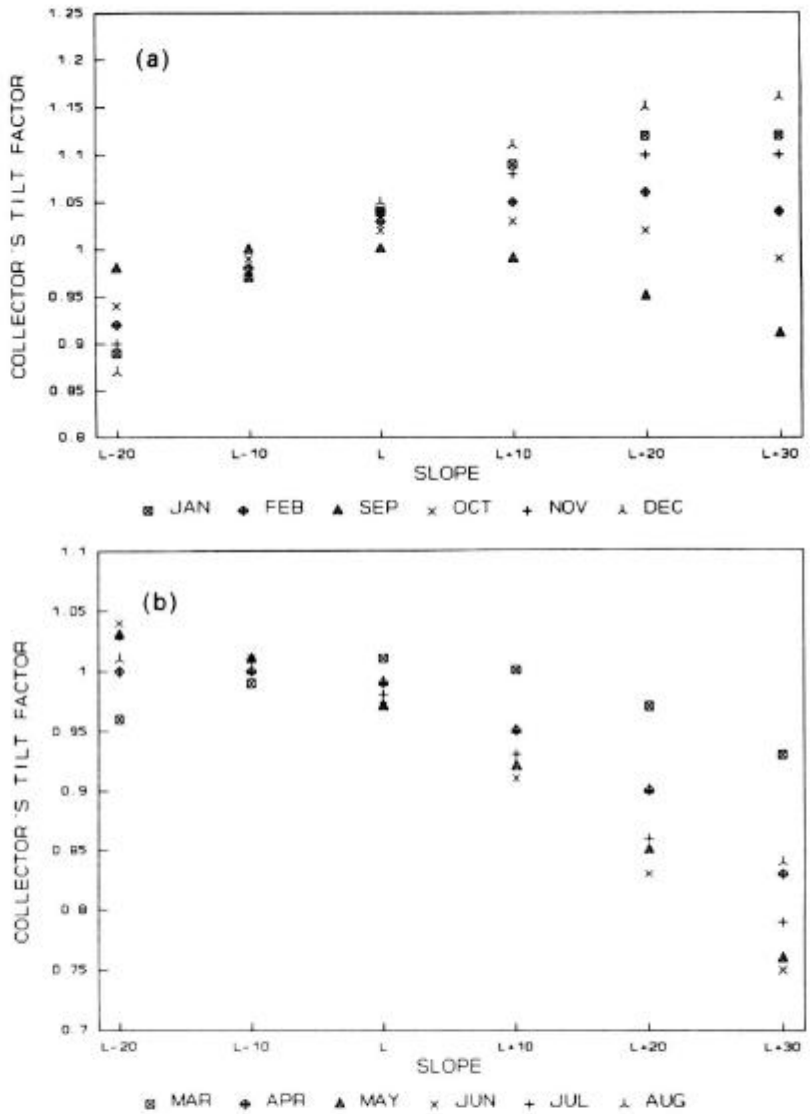

Fig. 7. Effect of inclination on collector's tilt factor for zone 1.

some transitional months may be observed between them. It is interesting to note that the solar-energy variations synchronize with typical climatological observations.

During the rainy season (i.e. April to August), insolation is at a maximum when the inclination equals (*L*–20) degrees. However, during the main dry season (i.e. October to February), insolation reaches the

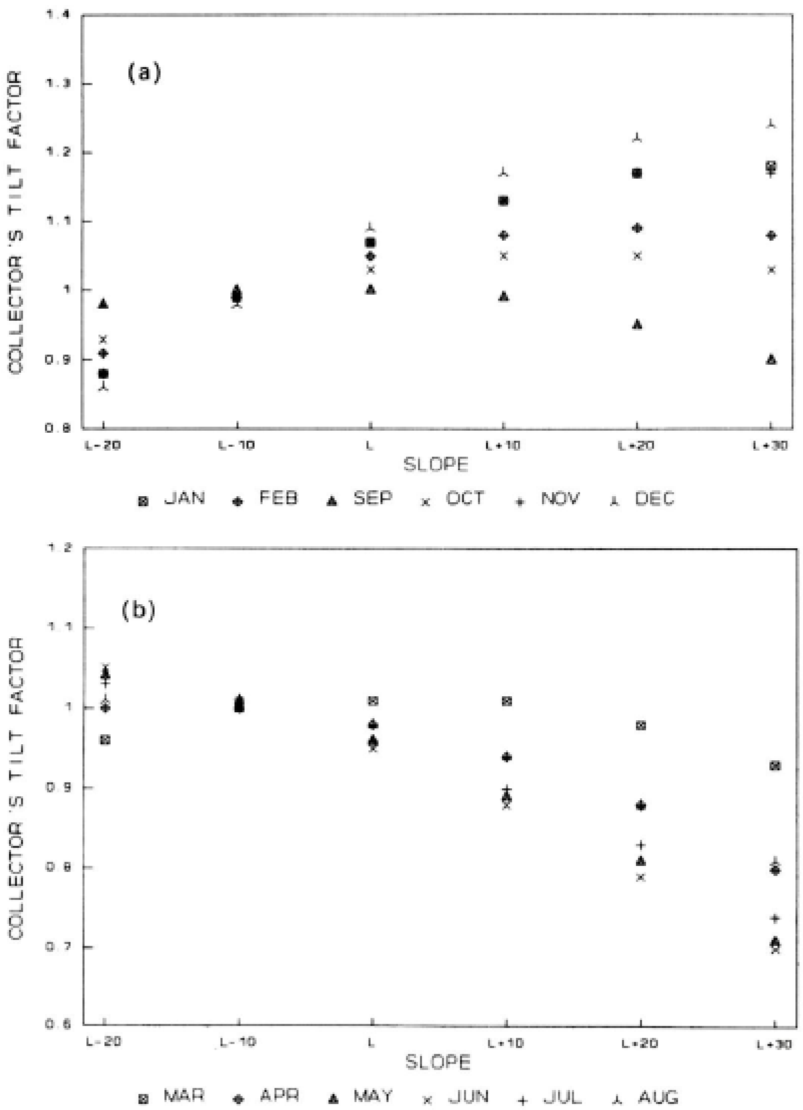

Fig. 8. Effect of inclination on collector's tilt factor for zone 2.

peak value when the surface is tilted between (L+10) and (L+30)degrees. March and September are transitional months.

In all cases, the total radiation received on an inclined surface is largest for all sites in zone 3. This zone lies mainly in the Sahara desert and is characterized by severe weather conditions including high temperatures for most of the year.

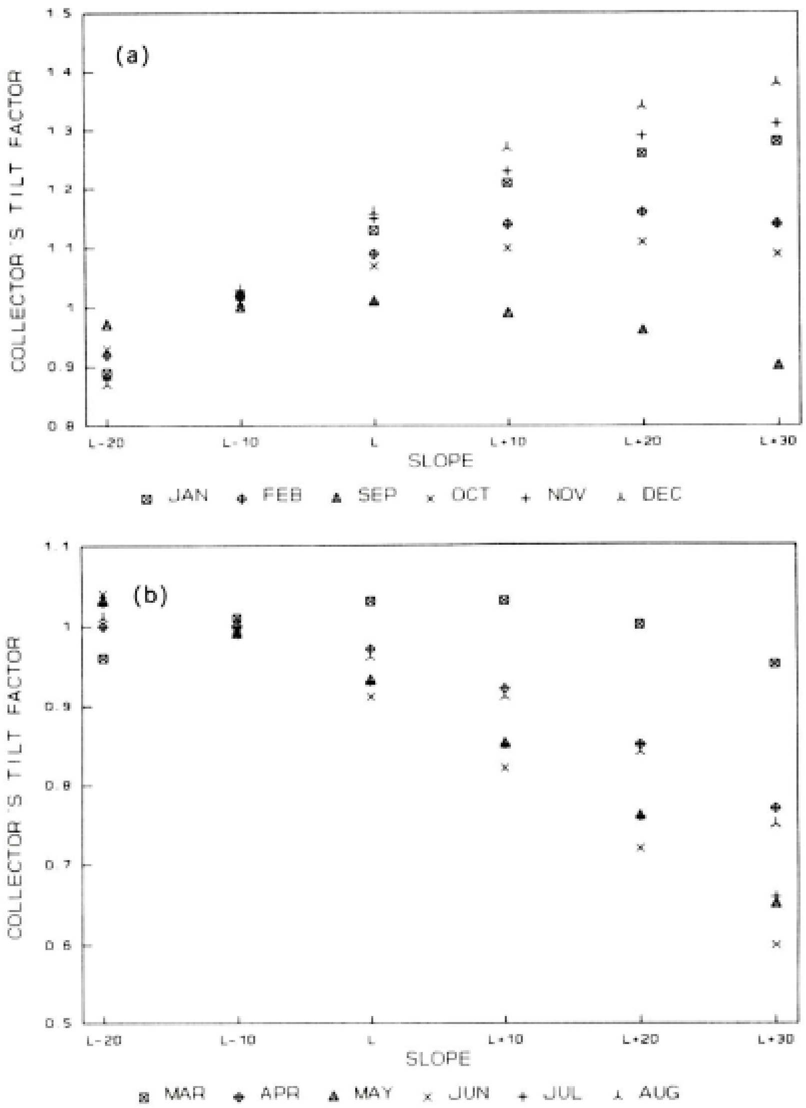

Fig. 9. Effect of inclination on collector's tilt factor for zone 3.

Figure 7 shows the variations of the collector's tilt factor with inclination for zone 1. Inclination is beneficial for a tilt factor of greater than unity. It can be observed in Fig. 7(a), that, from October to February an inclination varying between L and $(L+20)$ degrees produces a total radiation greater than the corresponding global radiation on the same surface when horizontal. The gain reaches a maximum of 15% approximately. In Fig. 7(b),

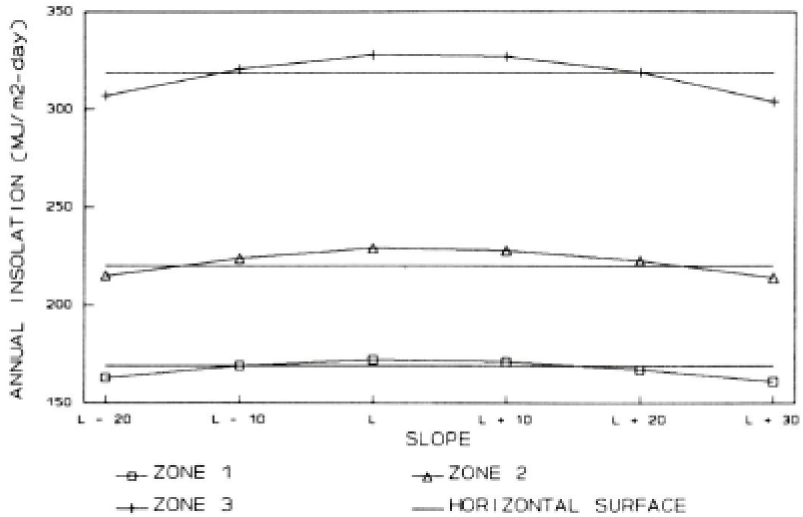

Fig. 10. Variation of annual total global radiation on inclined surfaces.

it can be seen that varying the inclination between (*L*–10) and (*L*–20) degrees is only slightly beneficial for the months of April to August. This gain does not usually exceed about 5%. For March and September, inclination at exactly the value of the latitude or horizontal positioning of the surface seems to produce maximum radiation. Zones 2 and 3 also display similar patterns as depicted in Figs 8 and 9. However, the gains are higher in these zones, reaching about 21 and 35% respectively.

Consequently, three solar-energy seasons have been identified in this study, namely the high solar-energy season, the low solar-energy season and the transitional months.

Figure 10 presents the total annual global radiation received for various collector inclinations. The corresponding values on horizontal surfaces for each zone are juxtaposed in the figure. Thus, the gain or loss that is achieved by inclination can easily be assessed. An inclination of *L* degrees produces the best all-year-round performance. However, collectors tilted between (*L*–10) and (*L*+10) degrees should produce satisfactory results. The maximum total yearly radiation varies from about 173 MJ/m^2 day in zone 1 to about 320 MJ/m^2 day in zone 3. Thus, application of solar devices should be more attractive as the latitude *increases*.

4 CONCLUSIONS

The terrestrial radiation incident on inclined surfaces has been determined for three zones in Nigeria. The results show that insolation is strongly dependent on climatological factors such as cloudiness of the sky. Attenuation of the extraterrestrial radiation by the atmosphere is significantly higher as the latitude is *decreased*.

Three solar energy seasons have been identified in this investigation and a unique inclination is ideal for each of these seasons. The maximum benefit arising from tilting the collector's surface varies from about 5 to 35%, depending on both the geographical location and time of year. However, it seems that for optimal all-year-round performance of solar systems, the collector surface should be fixed at an inclination between (L–10) and (L+10) degrees.

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