DEVELOPMENT OF A MATHEMATICAL MODEL FOR REACTIVE POWER TRANSMISSION COSTS IN ELECTRIC POWER SYSTEMS

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Abstract: The paper gives detailed analyses to help determine the increase in production costs due to the transmission of reactive power. On the basis of that and with due consideration to the cost of VARS compensation equipment, a mathematical model is developed to determine an economically justified transmission distance for VARS in electric power systems. Standard voltages and line parameters are used for the computations. MATLAB computer programming is used to obtain the numerical results. The developed mathematical model and the numerical results could be useful to electric power systems engineers.

Key words: Mathematical model, reactive power transmission costs, justified transmission distance.

1. INTRODUCTION

Electricity cannot be stored in large quantities and the power authority has little control over load demand. Power engineers maintain the balance between generated output and systems load demand at a specific voltage and frequency. The systems' load is made up of active and reactive power (also harmonics; however the parameter magnitudes involved are much smaller, and usually not dominant). Active power is generated at the sending end, while VARS may be either transmitted or compensated for at the consuming (receiving) end. But the transmission of reactive power increases the following losses:

- Active power losses (in the transmission systems). during the period of maximum load demand, which calls for an increase in the generated output;
- Energy losses (in the system), leading to unnecessary expenditure of fuel; and
- Changes in voltage levels in the system.

All these factors are taken into consideration to determine the economically justified distance for reactive power transmission. Basic engineering equations are used. Step by step approach is applied to obtain the final mathematical model.

2. ECONOMIC ANALYSIS

Transmission of both active and reactive power lead to losses in the system as mentioned in the introduction. Since active power is usually generated specifically to compensate for load demand, it is the reactive power that is controlled to achieve a reduction of losses in the system.

When a power system is being designed and the parameters are yet to be determined, it is a generally accepted must to compensate for the predicted reactive power demand at the consuming end so as to reduce losses in the system. This reduction of the total transmitted power allows for the use of smaller conductors for transmission, leading to the reduction of system construction costs. Because of the expensive nature of the compensation equipment, the cost is also taken into account in determining the most economically justified distance for reactive power transmission.

2.1. Added Increase in cost of systems equipment

The total current in any system element of a three-phase network is given as,

$$I = \frac{\sqrt{p^2 + q^2}}{\sqrt{3}V} = \frac{p\sqrt{1 + \tan^2 \Phi}}{\sqrt{3}V} \tag{1}$$

Where:

p = active power
q = reactive power

The cross section area of a power transmission conductor is given as,

$$F = \frac{I}{J} = \frac{p\sqrt{1+\tan^2\Phi}}{\sqrt{3}VJ} \qquad (2)$$

Where:

J is the current density of the conductor in A/mm^2

The total cost of transmission line per km due to the added losses is [1],

$$B_L = (b_{0L} + b_L F)L \qquad (3)$$

Where:

b_L – a variable constant reflecting increase in cost of conductor, Pula/km.mm^2.

b_{0L} – a fixed cost component of the conductor, Pula/km.

L ,km – total length of the conductor.

Substitution of equation (2) into equation (3) gives equation (4),

$$B_L = (b_{0L} + \frac{b_L p\sqrt{1+\tan^2\Phi}}{\sqrt{3}VJ})L \qquad (4)$$

The additional cost due to the transmission of apparent power as compared with the transmission of purely active power is expressed as,

$$\Delta B_L = \frac{b_L pL}{\sqrt{3}VJ}(\sqrt{1+\tan^2\Phi} - 1) \qquad (5)$$

Increased cost per unit of reactive power transmission is given by,

$$b_{La} = \frac{\Delta B_L}{p\tan\Phi} = \frac{b_L L(\sqrt{1+\tan^2\Phi} - 1)}{\sqrt{3}VJ\tan\Phi} \qquad (6)$$

VARS transmission increases the apparent power and hence the rating of transformers. The transformer rating of one transformer substation S_{T1} is,

$$S_{T1} = \sqrt{p^2 + q^2} = p\sqrt{1+\tan^2\Phi} \qquad (7)$$

$$S_{T2} = \frac{p\sqrt{1+\tan^2\Phi}}{1.4} \qquad (8)$$

(For a two – transformer substation S_{T2}, the rating of each transformer is approximately sixty percent of the total load, i.e. S/1.4) [2, 15]

Therefore the rise in cost per unit of reactive power transmission is, for a one-transformer substation,

$$b_{TU1} = \frac{b_T p(\sqrt{1+\tan^2\Phi} - 1)}{p\tan\Phi}$$

$$= \frac{b_T(\sqrt{1+\tan^2\Phi} - 1)}{\tan\Phi} \qquad (9)$$

and for a two transformer substation,

$$btu2 = \frac{b_T(\sqrt{1+\tan^2\Phi} - 1)}{1.4\tan\Phi} \qquad (10)$$

2.2. Losses due to reactive power transmission

Active power loss: Active power loss in the line is

$$\Delta P_L = \frac{p^2 + q^2}{V^2}R_L = \frac{p^2}{V^2}(1+\tan^2\Phi)R_L \quad (11)$$

For a balanced active power in the system, the generated output at the power station should be increased to meet the extra active power loss due to the transmission of VARS. Such an increase in the generated output is considered economically permissible if the cost due to the additional power loss does not exceed the cost of installing and maintaining the compensating VARS equipment at the consuming end [3, 19], i.e.

$$K_a \Delta P_L \le K_r Q_r \tag{12}$$

Where:

K_a = Cost / kW of generated output, Pula / kW,

K_r = Cost / kVAr of VARS compensating equipment, Pula / kVAr,

Q_r = kVAr rating of reactive power equipment, kVAr.

Reactive power loss: Reactive power transmission leads to voltage drop in transmission lines. The reactive power loss is,

$$\Delta Q_L = \frac{p^2}{V^2}(1 - \tan^2 \Phi) X_o L \tag{13}$$

Where:

X_o = unit reactance of the line, Ω / km.

This loss is taken into account in the reactive power balance in the system. As such, the installed VARS source in the system should be increased to compensate for the loss. (Note that VARS transmission from the generator has technical constraints). VARS transmission is considered economical if the cost of generation at the power station, (including losses in the system) is less than or equal to the cost (excluding losses in the system) of installing VARS compensating equipment at the consuming end [3,4], i.e.

$$K_Q (Q_{beg} + \Delta Q_L) \le K_r Q_r \tag{14}$$

Where:

K_Q = energy loss due to VARS transmission, kWh,

Q_{beg} = VARS output at the beginning of the line, kVAr.

Equations (13) and (14) are used to establish approximately the economic justifiable distance for VARS transmission (considering only power loss) as,

$$L_r = \frac{(K_r - K_Q)V^2 p \tan\Phi}{X_o K_Q p^2 (1 + \tan^2 \Phi)} \approx \frac{(K_r - K_Q)V^2}{X_o K_Q p \tan\Phi} \tag{15}$$

Energy loss: Energy loss is expressed as,

$$\Delta A_Q = \Delta P_Q \xi_Q \tag{16}$$

Where:

ξ_Q = average time / year, corresponding to the total time for VARS transmission, hr.

The energy loss leads to an increase in the use of fuel at the generating station. This increase in operational cost is obtained as [2, 3, 4],

$$C_F = \beta \sigma \Delta A_Q = \beta \sigma \xi_Q \frac{p^2}{V^2} \tan^2 \Phi R_L \tag{17}$$

Where:

β = cost of fuel, Pula/m^3,

σ = cubic metre of extra fuel used due to the compensation for the transmission of reactive power.

The cross-sectional area of the conductor is,

$$F = \frac{p}{\sqrt{3} V J \cos\Phi} = \rho \frac{L}{R_L} \tag{18}$$

From which,

$$R_L = \frac{\rho L \sqrt{3} J \cos\Phi}{p} \tag{19}$$

Where:

ρ = resistivity of conductor, Ω mm^2 / km

Hence,

$$C_F = \beta\sigma\xi_Q \frac{p}{V}\tan^2\Phi\rho L\sqrt{3}J\cos\Phi \quad (20)$$

Systems cost is given by [4, 17],

$$C = \alpha K_r Q_r + C_F \qquad (21)$$

Where:

α = a depreciation factor of the VARS compensation equipment.

The first component of equation (21) reflects capital cost and the second component, operational cost.

From equations (20) and (21),

$$C = \alpha K_r Q_r + \beta\sigma\xi_Q \frac{p}{V^2}\tan^2\Phi\rho L\sqrt{3}J\cos\Phi \qquad (22)$$

The additional cost is minimum if,

$$\frac{\partial C}{\partial(\tan\Phi)} = 0$$

Applying partial differentiation with respect to tanΦ, the approximate distance for VARS transmission (considering minimum losses in the system) is obtained as,

$$L_e = \frac{\alpha K_r V_{end}}{\beta\sigma\xi_Q\rho\sqrt{3}J(2\sin\Phi - \sin^3\Phi)} \qquad 23)$$

Where:

V_{end} = Voltage at the end of the line, kV.

If the voltage drop, due to VARS transmission is considered then,

$$L_e = \frac{\alpha K_r V_{beg} V_{nom}}{V_{nom}\beta\sigma\xi_Q\rho\sqrt{3}J(2\sin\Phi - \sin^3\Phi) + \alpha K_r p X_o \tan\Phi} \qquad (24)$$

Where:

V_{beg} = voltage at the beginning of the line,

V_{nom} = nominal voltage, kV.

The voltage at the end of the line in equation (23) can be expressed as,

$$V_{end} \equiv V_{beg} - \frac{QX_o L}{V_{nom}} = V_{beg} - \frac{p\tan\Phi X_o L}{V_{nom}} \qquad (25)$$

2.3. Economically justified distance for reactive power transmission

All the factors presented in sections 2.1 and 2.2 are considered to establish the total cost, C_T, due to the transmission of VARS.

$$C_T = [\Delta B_L + K_a \Delta P_L + K_Q(Q_{beg} + \Delta Q_L)] + C_F \qquad (26)$$

Substitution of appropriate values from equations (5), (11), (14) and (20) into equation (26) expands C_T into,

$$\begin{aligned} C_T = {} & \alpha[\frac{b_L pL}{\sqrt{3}VJ}(\sqrt{1+\tan^2\Phi} - 1) + \\ & K_a \frac{p}{V}(1+\tan^2\Phi)\rho L\sqrt{3}J\cos\Phi + \\ & K_Q(P\tan\Phi + \frac{P^2}{V^2}\tan^2\Phi X_o L)] + \\ & \beta\sigma\xi_Q \frac{P}{V}\tan^2\Phi\rho L\sqrt{3}J\cos\Phi \qquad (27) \end{aligned}$$

For VARS transmission to be economically justified, C_T must be minimum, i.e.

$$\begin{aligned}\frac{\partial C_T}{\partial(\tan\Phi)} &= \frac{\alpha b_L PL}{\sqrt{3}JV}\frac{\tan\Phi}{\sqrt{1+\tan^2\Phi}} + \\ &\alpha K_a \rho L\sqrt{3}J\frac{P}{V}(2\sin\Phi - \sin^3\Phi) + \\ &\alpha K_Q P + \alpha K_Q \frac{P^2}{V^2} 2\tan\Phi X_o L + \\ &\beta\sigma\xi_Q \rho L\sqrt{3}J\frac{P}{V}(2\sin\Phi - \sin^3\Phi) = 0\end{aligned} \tag{28}$$

The economically justified distance for VARS transmission is obtained from equation (28) as,

$$\begin{aligned}L_{opt} &= \frac{\alpha K_Q V^2}{\alpha\tan\Phi(\frac{b_L V}{\sqrt{3}J\sqrt{1+\tan^2\Phi}} + 2K_Q P X_o)} + \\ &+ \sqrt{3}\rho JV(2\sin\Phi + \sin^3\Phi)(\alpha K_a + \beta\sigma\xi_Q)\end{aligned} \tag{29}$$

Equation (29) is the mathematical model, which is used to compute the economically justified distance for VARS transmission.

3. RESULTS AND ANALYSIS

The system flow chart used in the implementation of the equation (29) as shown in Figure 1. The program was written using the computing environment MATLAB [5, 6, 20]

The MATLAB computer results in Table 2 and Figure 2 are obtained on the basis of the following data presented in Table 1 (The data represents nominal voltages, and the corresponding values of maximum power that can be transmitted and the increase in cost of conductor due to the transmission of reactive power) [7, 8, 9, 10]

Table 1: Data used to compute the results of VARS transmission

V, kV	33	66	110	132	220	330
P, MW	10	30	60	75	100	130
bL, Pula / MVAr	9.8	10.2	10.7	11	11.5	12.5

Where:

$\alpha = 0.17$,

$\beta\sigma = 0.01$ Pula / kWh,

$\rho = 29\Omega$ mm^2

$\xi_Q = 1000$ hr

$K_Q = 1.6$ Pula / kVAr,

$K_a = 10$ Pula / kW,

$J = 1.0$ A / mm^2,

$X_o = 0.4\ \Omega$ / mm^2.

Table 2: Economically justified distance for VARS transmission

V (kV)	33	66	110	132	220	330
tan(φ)	Lopt. Km	Lopt. Km	Lopt. Km	Lopt. Km	Lopt. Km	Lopt. Km
1.00	3.0731	6.0875	10.0639	12.0440	20.0485	29.8658
0.95	3.2314	6.4028	10.5864	12.6694	21.0827	31.8977
0.90	3.4131	6.7642	11.1847	13.3855	22.2678	33.1539
0.85	3.6218	7.1789	11.8709	14.2068	23.6279	35.1700
0.80	3.8621	6.6561	12.6601	15.1512	25.1925	37.4898
0.75	4.1398	8.2070	13.5711	16.2411	26.9989	40.1684
0.70	4.4620	8.8462	14.6276	17.5052	29.0945	43.2761
0.65	4.8383	9.5924	15.8609	18.9807	31.5409	46.9044
0.60	5.2812	10.4706	17.3121	20.7167	34.4198	51.1744
0.55	5.8078	11.5145	19.0871	22.7803	37.8421	56.2507
0.50	6.4420	12.7714	21.1138	25.2646	41.9627	62.3631
0.45	7.2180	14.3093	23.6547	28.3041	47.0047	69.8428
0.40	8.1872	16.2302	26.8283	32.1005	53.3027	79.1864
0.35	9.4306	18.6943	30.8992	36.9704	61.3820	91.1739
0.30	11.0827	21.9684	36.3085	43.4414	72.1187	107.1055
0.25	13.3863	26.5334	43.8509	52.4642	87.0909	129.3230

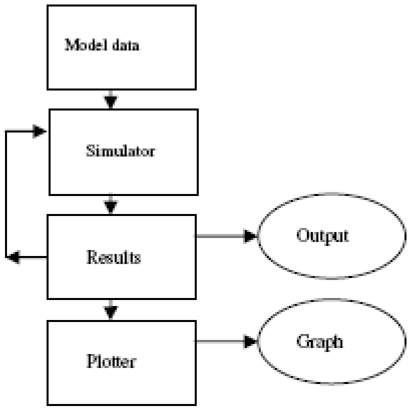

Fig. 1. System flowchart

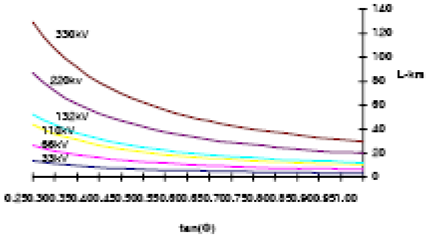

Fig. 2. Graph for VARS transmission

4. CONCLUSION

The results show that,

- VARS compensations equipment should optimally not be installed at the consuming end of the network,
- There is a distance for which VARS transmission can be considered economical: (table 2 and figure 2)
- The graph of tanφ vs. transmission distance is inverse. For a specified voltage, the smaller the reactive power, the longer the economically justified distance.

The cost of electric equipment is high and continues to rise yearly. It is expected of power systems engineers to design systems that are not only reliable and stable, but also can operate economically. In this respect, a number of designs have to be considered and compared before a final project is selected.

This paper has established a mathematical model that could be used to help determine the economically justified distance for VARS transmission. The computer results are satisfactory and could be of guidance to consulting and power systems design engineers who have to justify their projects technically and economically, especially in choosing conductor and equipment sizes / ratings.

5. REFERENCES

[1] N. A. Manlike: "Reactive Power Compensation in Power Systems" Energy Journal, Moscow. (1983).

[2] M. I. Aberson: "Compensation of Reactive Power in Modern Power Systems" IEEE Journal, London. (1989).

[3] B. N. Neklepayev: *Elecric Stations and Systems Reference Book*: High School Publishers, Moscow. (1982).

[4] N. A. Kasak: "Technico – Economic Calculations of Reactive Power Compensation in Networks" Electrichestvo Journal, Moscow. (1985).

[5] *The Ultimate Computing Environment "MATLAB"*, Prentice Hall, Englewood Cliffs, NJ 07632, (1995).

[6] M. Carnegie: *Control Tutorial for Matlab – Animations & Command*, The Regents of the University of Michigan. (1996).

[7] G. O. Anderson: "Development of Mathematical Model for VARS Transmission in Electric Power Systems". *IEEE Africon (2002),* George, South Africa Vol. 2 Pages 941-945.

[8] G. O. Anderson: "Computer Simulation of Reactive Power Compensation, a Linear Programming Approach" International Journal of Power and Energy Systems, Vol 16, No.2, IASTED, Canada. (1996). pp. 78-82.

[9] G. O. Anderson: "Economic Dispatch of Active Power from Modern Power Systems", *1st Africa-USA Conference on Manufacturing Technology*, Lagos, Nigeria (1992) pp. 401-410.

[10] P. Pohjanheimo, E. Lakervi: "A Simplified Steady State Model for a Dynamic Voltage

Restorer" *Proceedings of IASTED: International Conference on Power and Energy Systems*. Nov. 8 – 10, 1999. Las Vegas, USA. pp. 278-284.

[11] Sayaoad Hameida: "A New Approach for Optimal Power Dispatch Problem". *Proceedings of the IASTED : International Conference on Power and Energy Systems.* Nov 8 – 10, 1999. Las Vegas, USA. pp. 509-515

[12] F. T. Sparror: "Optimal Placement of FACTS Devices in Multiple Time Periods and Variable Load Conditions". *Proceedings of the IASTED : International Conference on Power and Energy Systems.* Nov 8 – 10, 1999. Las Vegas, USA. pp. 89 - 93

[13] B. Vankatesh : "Voltage Improvement of Distribution Networks by proposing Alternate Radial Distribution Systems". *Proceedings of the IASTED International Conference on Power and Energy Systems.* Nov 19 – 22, 2001. Tampa, Florida, USA. pp. 374 - 377

[14] Jos Carlos de Olivereira: "Advanced Static VARS Compensation Performance under Non-Ideal Electrical Power Supply Conditions". *Proceedings of the IEEE : Ninth International Conference on Harmonics and Power Quality*. Volume II. October 1 – 4, 2000. Orlando, Florida, USA. pp. 554 – 559.

[15] B. Vijay, P. Bodger: A. Wood, "Towards a Practical Partial Core Transformer- Compensation of Reactive Power Requirements with a VSC" *Proceedings of the Second IASTED Africa Conference on Power and Energy Systems.* September 8-10, 2008. Gaborone, Botswana, pp 32-37

[16] J. Doval, C. M. Penalver, A. Lago: "Consideration of DSP controlled VSC based STATCOM". *IEEE International Symposium on Industrial Electronics*, 9-11 June2003, pp 397-400.

[17] Y. galu, L. Munda, A. A. Jimoh: "Sability Analysis of Centurion Electric Power System", *Proceedings of the Second IASTED Africa Conference on Power and Energy Systems.* September 8-10, 2008. Gaborone, Botswana, pp 51-56.

[18] K. R. Padiyar: *Power Systems Stability Dynamics and control,* BS Publications, 2nd Edition, Hyderabad, India, 2008.

[19] D. T. Oyedokun, K. A. Folly: "Power Flow Studies in HVAC and HVDC Transmission Lines", *Proceedings of the Second IASTED Africa Conference on Power and Energy Systems.* September 8-10, 2008. Gaborone, Botswana, pp 57-63.

[20] J. Novak, P. Chalupa, V. Bobal: "Power Balance Control Using Evolutionary Algorithm", *Proceedings of the Second IASTED Africa Conference on Power and Energy Systems.* September 8-10, 2008. Gaborone, Botswana, pp 207-211.

[21] P. Havel, O. Novak, J. Frantik: "Optimal Range of Regulation Reserves for Power Balance Control in the Electric Grid", *Proceedings of the 16th International Conference on Process Control*, Strbske Pleso, Slovakia, 2007.