APPLICATION OF THE METHOD OF D-PARTITIONING FOR STABILITY OF CONTROL SYSTEMS WITH VARIABLE PARAMETERS

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The control system stability could be determined by applying any one of the well-known stability criterions: Nyquist, Bode, Hurwitz, Routh, etc., only if the system's parameters are defined and constant. When the objective is to explore the influence of variations of a system's parameter on its stability, then one of the stability criterions has to be applied repeatedly. This implies that calculations should be completed for each individual case of repetition. To save time and effort and to analyze the effects of parameters variation on the system's stability, a D-partitioning method is applied. It could be implemented when one, two or more system's parameters are varied. The method employs the possibility to define regions of stability in the space of the system's parameters. The D-partitioning method is essential and very useful when analyzing robust control systems. A research on the application of the method has the objective to clarify it in a popular manner and simplify its implementation. By examining a number of different control systems with variable parameters and applying the method of the D-partitioning, useful results are achieved that contribute further to stability theory of control systems.

***Keywords:* Control systems, criterion, regions of stability, instability, variable parameters, D-partitioning, space charecteristics, equation coefficients, special lines, gain, time constant**

1 INTRODUCTION

By applying any one of the known stability criterions like, Nyquist, Bode, Hurwitz, Routh or Raus the stability of a control system can be determined only if the system's parameters are defined and constant. To determine the stability when one or more of the system's parameters are variable, some of the stability criterions have to be applied repeatedly. Then all procedures should be implemented for each individual case of repetition. This problem could be eventually overcome if specialized software like CODAS is designed for atomizing the mentioned criterions. It may go over all the procedures automatically and determine the regions of stability corresponding to the variation of a specific system parameter.

There are a number of known methods and criterions dealing with the problem of stability of systems with variable parameters. Unfortunately some of them are limited in their application, while the implementations of others require special technical skills. For instance, the criterion of Vishegradsky [1,2] is limited to third order control systems and it is restricted to variation of only one parameter. It only defines the regions of stability or instability if the gain of the system is changed. Further, the Root-Locus technique, developed by Evans [3], displays the trajectories of the poles of a system (the root loci) when a certain system parameter varies. For plotting the root loci accurately, usually computer programs like the ROOTLOCI in the ACSP package, or the RIPLOT in the CSAD/MATLAB toolbox, or the ROOT LOCUS of the program CC are required [4,5]. Although this technique is described in a lot of control engineering sources, in order to be implemented, it needs an appropriate computer software as well as high personal experience for a proper interpretation of the data provided by the root loci analysis. Another well-known criterion for analysis of multi-loop high order systems with variable parameters is the criterion of Mikhailov [1,2,6]. It can be successfully used for determination the stability of such systems, but is quite sophisticated and again needs special software.

In order to save effort and time and to analyze the effects of parameter variations on the system's stability, the method of the D-partitioning can be applied. The initial ideas of the method were suggested by Neimark [1,2]. Regrettably these ideas were not sufficiently developed for any broader practical implementation. The ideas of the method have been rarely described, if at all, in any sources on control engineering and have never being implemented in practice because of their obscurity and complexity [2,6,7].

If the method of the D-partitioning is well clarified and simplified, it has advantages compared with the other mentioned methods in terms of a clear and graphical display of all regions of variation of each parameter for which the system remains stable.

The purpose of the current research is to emphasize on the application of the method and has the objective to clarify it in a popular manner in order to simplify its implementation. By applying the basic initial ideas of the method, the main line of the mathematical approach and the suggested examples in this paper are originally designed by the authors. The contribution of this research is to simplify and graphically determine the regions of stability for variation of specific systems parameters. The method of the D-partitioning is applied to control systems with different variable parameters and some useful results are achieved, contributing further to the theory of control systems stability. With the aid of the method proper parameter values can be chosen for a desirable performance and stability of a system. The method of the D-partitioning can be practically used when one, two or more system's parameters are varied. Basically the method defines regions of stability in the space of the system's parameters. The D-partitioning method can be essential and very useful when analyzing robust control systems.

2 NATURE OF THE D- PARTITIONING

2.1 Space of Characteristic Equation Coefficients

The roots of the characteristic equation of the transfer function of a control system depend on the coefficients of this equation and therefore depend on the system's parameters. Generally, for an n-order characteristic equation, m roots may be positioned in the right-hand side and $(n-m)$ in the left-hand side of the s-plane [7,9].

To implement the method of the D-partitioning, an n-order characteristic equation is presented in the format:

$$G(s) = a_0 s^n + a_1 s^{n-1} + ... + a_n = 0 \quad (1)$$

where s is the Laplace operator

$a_0, a_1, ..., a_n$ - parameter-dependent coefficients

If $a_0 = 1$ and the position of one of the roots is at the origin of the coordinate system, or a pair of roots is at the imaginary axis (marginal case of stability), after substituting $s = j\omega$, equation (1) can be transformed to equation (2):

$$G(j\omega) = (j\omega)^n + a_1 (j\omega)^{n-1} + ... + a_n = 0 \quad (2)$$

If the frequency changes within the range of $-\infty \le \omega \le +\infty$ in the space of the characteristic equation coefficients $a_1, a_2, ..., a_{n-1}, a_n$, equation (2) represents a plane in this n-dimensional space.

When the characteristic equation is of a third order, it can be described by equation (3).

$$G(s) = s^3 + a_1 s^2 + a_2 s + a_3 = 0 \quad (3)$$

There is always such a correlation between the coefficients of equation (3) at which either the position of one of the roots is at the origin of the coordinate system, or a pair of roots is placed on the imaginary axis of the s-plane. Then, after substituting $s = j\omega$ in equation (3), equation (4) is obtained, defining a plane N in the 3-dimensional space of the coefficients: a_1, a_2, a_3, as shown in Figure 1 [1,2,8,9].

$$G(j\omega) = (j\omega)^3 + a_1 (j\omega)^2 + a_2 (j\omega) + a_3 = 0 \quad (4)$$

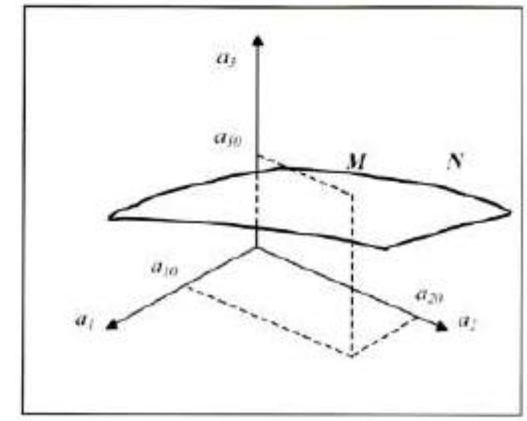

Figure 1. The surface N in the 3-dimensional space of the characteristic equation coefficients

Specific values of the coefficients: a_{10}, a_{20}, a_{30} define the point M, as seen in Figure 1, that will be positioned on the surface N only when the roots of the characteristic equation are on the imaginary axis (the marginal case of stability). Therefore the surface N, defined by equation (4), divides the space of the coefficients into a number of regions. These regions are labeled by $D(m)$, where m is the number of the roots in the right-hand side of the s-plane. Each point into a region corresponds to specific system's parameters and hence to a specific third order characteristic equation that has specific number of roots m in the right hand-side of the s-plane [7,8,9].

2.2 Definition and Further Considerations on the D-partitioning

The division of the space of the characteristic equation coefficients $a_1, a_2, ..., a_{n-1}, a_n$, *into a number of regions corresponding to different number of the roots* m *in the right-hand side of the s-plane is considered as the D-partitioning.*[1,8,9]

In case of a third order system, described by the characteristic equation (3), there will be four regions: *D(3), D(2), D(1), D(0)*.

Only the region *D(0)* will be a region of stability, since only it corresponds to such a correlation of the coefficients, respectively of the system's parameters, for which the characteristic equation has no roots in the right hand-side of the s-plane. Therefore, the condition for stability is met only for region *D(0)*.

Any crossing from one to another region of the space of the coefficients $a_1, a_2,... a_n$ corresponds to passing over of the characteristic equation roots across the imaginary axis of the s-plane. Therefore, the borders of the D-partitioning can be determined by substituting $s = j\omega$ in the characteristic equation and further by allocating different values of the frequency ω, calculating the parameter coordinates [2,8,9].

If the characteristic equation is of n^{th} order, an n-dimensional space is considered instead of a 3-dimensional space. But then, in most of the cases, only some of the system's parameters and hence only some of the coefficients may be variable. For instance, in a third order system, only the coefficients a_1, a_2 may be variable, while a_3 may be a constant. Then, instead of a surface dividing the space of the coefficients, only a curve is considered. This curve is obtained as a cross section of the surface N with the plane $a_3 =$ const.

There is a considerable theoretical and practical significance of the method of the D-partitioning. But its implementation for higher order systems, with larger number of parameter variables, is becoming quite complex and is difficult to be presented graphically. Practically the method of the D-partitioning could be applied mainly for cases of control systems with one or two simultaneously varied parameters.

3 D-PARTITIONING BY ONE VARIABLE SYSTEM PARAMETER

3.1 General Application of the D-Partitioning by One Variable System Parameter

The system characteristic equation (1) can be presented in the following format to expose the variable parameter [2,10]:

$$G(s) = P(s) + vQ(s) = 0, \qquad (5)$$

where $P(s)$ and $Q(s)$ are polynomials of s
v is the variable system parameter

The D-partitioning regions could be obtained by substituting the Laplace operator $s = j\omega$. Then, equation (6) becomes a complex number equation

$$G(j\omega) = P(j\omega) + vQ(j\omega) = 0 \qquad (6)$$

Therefore, v can be presented also as a complex number as follows:

$$v = -\frac{P(j\omega)}{Q(j\omega)} = X(\omega) + jY(\omega), \qquad (7)$$

where the real part $X(\omega)$ of this complex number corresponds in reality after substituting $s = j\omega$ to the variable parameter of the control system.

The D-partitioning regions could be obtained graphically in the plane $v = X(\omega) + jY(\omega)$, by allocating different values of the frequency within the range $-\infty \le \omega \le +\infty$. It is quite sufficient to plot the border of the D-partitioning regions only within the range $\omega \le +\infty$, since it could be complemented for the range $-\infty \le \omega$ by its mirror image with respect to the real axis.

Suppose a root of the system's characteristic equation, is moved from the right-hand side to the left-hand side of the s-plane, crossing the imaginary axis, as shown in Figure 2 [2,11].

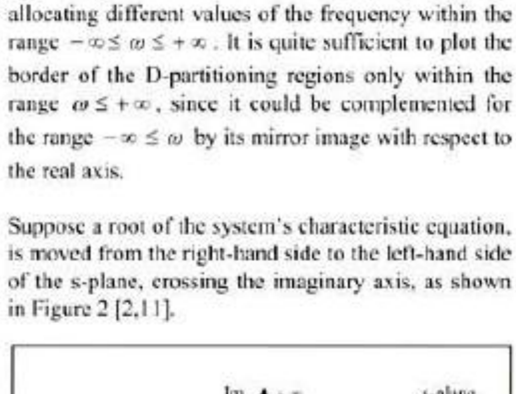

Figure 2. Root movement from the stable into the instable region of the s-plane.

This movement of the root from the unstable into the stable region of the s-plane corresponds to a crossing of the border of the D-partitioning region in a specific direction, seen in Figure 3.

If a root is moved along the imaginary axis of the s-plane when the frequency ω changes from $-\infty$ to $+\infty$, the region of stability remains on the left-hand side of the plane (Fig.2). Similarly, in the complex plane $v = X(\omega) + jY(\omega)$, the region of stability remains on the left-hand side of the D-partitioning curve for a change of frequency from $-\infty$ to $+\infty$ (Figure3).

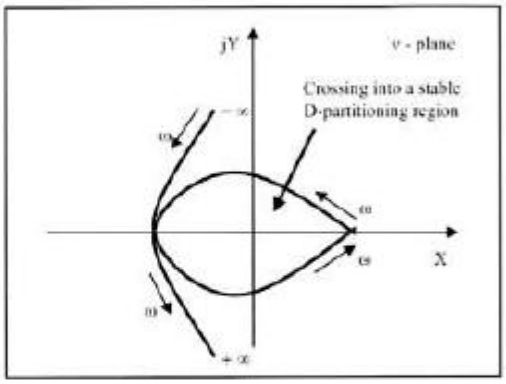

Figure 3. A D-partitioning curve obtained in the ν-plane for e frequency change from $-\infty$ to $+\infty$.

3.2 First Example of D-partitioning by One System Parameter (Gain Variable)

In this research, a number of original examples for practical implementation of the method are shown. Suppose that the characteristic equation of a system is:

$$G(s) = (T_1 s + 1)(T_2 s + 1)(T_3 s + 1) + K = 0 \qquad (8)$$

Any one of the system's parameters may be chosen as a variable. In this case, the time-constants T_1, T_2 and T_3 are known and constant values, while the gain $K = \nu$ is considered as the unknown and variable parameter. The objective is, by implementing th D-partitioning method, to determine the region of stability when the system parameter K varies and to determine the limits of K within which the control system will remain stable.

By applying equation (5), the polynomials $P(s)$ and $Q(s)$ could be determined as:

$$P(s) = (T_1 s + 1)(T_2 s + 1)(T_3 s + 1) \qquad (9)$$

$$Q(s) = 1 \qquad (10)$$

Substituting $s = j\omega$ in (9) and (10), applying equation (7) and considering that $K = \nu$, the D-partitioning curve (Figure 4) can be plotted from equation (11) by allocating values of the frequency from $-\infty$ to $+\infty$.

$$\left.\begin{aligned} \nu = K &= -\frac{P(j\omega)}{Q(j\omega)} \\ &= -(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_3) \\ &= X(\omega) + jY(\omega) \end{aligned}\right\} \qquad (11)$$

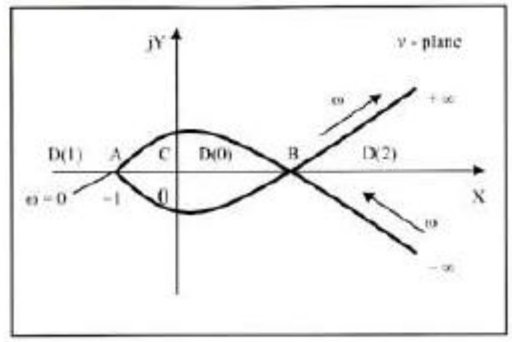

Figure 4. The D-partitioning curve defining three regions (partitions): D(0), D(1) and D(2).

As seen in Figure 4, the D-partitioning determines three regions on the ν-plane: D(0), D(1) and D(2). Only D(0) is the region of stability, since it is the one being on the left-hand side of the curve for a frequency change from $-\infty$ to $+\infty$.

In this case, the ν-plane is considered also as a K-plane, because $\nu = K$. To determine the range of K for which the system is stable it is required to take into account only the real values of K, since only the real values may be allocated to the gain of the system. If $K = 0$, corresponding to a point C on the ν-plane with coordinates (0, j0), the characteristic equation (8) is converted into:

$$G(s) = (T_1 s + 1)(T_2 s + 1)(T_3 s + 1) = 0 \qquad (12)$$

All three roots of equation (12) have negative real parts, being in the left-hand side of the s-plane:

$$s_1 = -\frac{1}{T_1}, \quad s_2 = -\frac{1}{T_2}, \quad s_3 = -\frac{1}{T_3} \qquad (13)$$

This also proves that point C(0, j0) corresponding to $K = 0$ belongs to the stable region D(0).

Further, it is obvious that if the parameter K is varied within the range of values, corresponding to the segment AB, the system will be stable, since the segment AB is within the stable region D(0).

Point A(-1, j0) determines the value of the gain as $K = -1$. When the gain is negative and within the range $-1 \geq K \geq 0$, the control system employs a positive feedback.

Point B(K_{max}, j0) corresponds to the maximum value of the gain, K_{max}, at which the system is reaching its margin of stability, or becomes marginal.

For the cases of process control systems that mainly employ negative feedback, the range of K is $0 \geq K \geq K_{max}$, corresponding to the segment CB. In reality, also proper phase and gain margins should be considered.

The value of K_{max} can be determined by finding out the frequency ω_0 at which the imaginary part $Y(\omega_0)$ of equation (11) is equal to zero, or $Y(\omega_0) = 0$. This frequency is determined as:

$$\omega_0 = \sqrt{\frac{T_1 + T_2 + T_3}{T_1 T_2 T_3}} \qquad (14)$$

Then after substituting $\omega = \omega_0$ into the real part of equation (11), the marginal system gain is determined as:

$$\left.\begin{aligned} K_{max} &= X(\omega_0) = \\ &= \left(1 + \frac{T_2}{T_1} + \frac{T_3}{T_1}\right)\left(1 + \frac{T_1}{T_2} + \frac{T_1}{T_3}\right) - 1 \end{aligned}\right\} \qquad (15)$$

3.3 Second Example of D-partitioning by One System Parameter (Variable Time-constant)

Suppose, the same control system is considered, like the one in the first example. Its characteristic equation is described by equation (8), but now the time-constant $T_3 = \nu$ is unknown and variable, while all other system parameters are known and constant.

Here the objective is to determine the regions of stability if the system parameter T_3 varies. Similarly, the limits of T_3 should be determined, within which the control system will remain stable.

For this case:

$$G(s) = P(s) + \nu Q(s) = P(s) + T_3 Q(s) = 0 \qquad (16)$$

Considering equations (9) and (10) and taking into account that $T_3 = \nu$, the D-partitioning curve can be plotted from equation (17) by allocating frequency values within the range $-\infty \leq \omega \leq +\infty$ (Fig. 5).

$$\left.\begin{aligned} \nu = T_3 &= -\frac{P(j\omega)}{Q(j\omega)} \\ &= X(\omega) + jY(\omega) \end{aligned}\right\} \qquad (17)$$

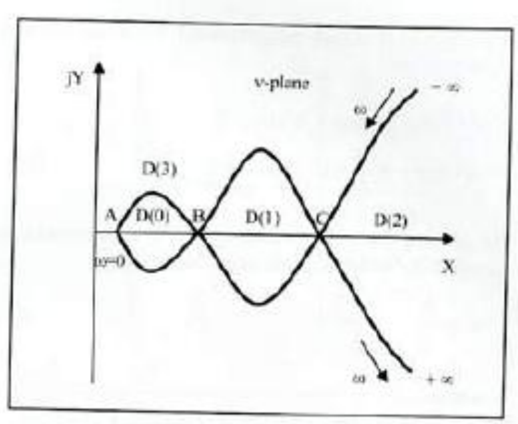

Figure 5. The D-partitioning defining four regions: D(0), D(1), D(2) and D(3)

As seen from Figure 5, the D-partitioning curve determines four regions on the ν-plane: D(0), D(1), D(2) and D(3). Here D(0) and D(2) are the regions of stability, since they are on the left-hand side of the curve for a frequency change from $-\infty$ to $+\infty$.

To find out the ranges of the time-constant T_3 for which the control system will be stable, a similar procedure as that in example one is applied. Here the coordinates of points A, B and C should be determined.

4 D-PARTITIONING BY TWO VARIABLE SYSTEM PARAMETERS

4.1 General Application of the D-Partitioning by Two Variable System Parameters

If two of the system parameters are variable simultaneously [2,7,12], the system characteristic equation (1) can be presented as:

$$G(s) = \mu P(s) + \gamma Q(s) + R(s) = 0\,, \qquad (18)$$

where $P(s)$, $Q(s)$ and $R(s)$ are polynomials of s; μ and γ are variables equal to the system's variable parameters

The border of the D-partitioning in the plain (μ, γ) is determined by:

$$G(j\omega) = \mu P(j\omega) + \gamma Q(j\omega) + R(j\omega) = 0 \qquad (19)$$

$$\text{If} \quad \left.\begin{aligned} P(j\omega) &= P_1(\omega) + jP_2(\omega) \\ Q(j\omega) &= Q_1(\omega) + jQ_2(\omega) \\ R(j\omega) &= R_1(\omega) + jR_2(\omega) \end{aligned}\right\} \qquad (20)$$

Equation (19) could be presented by a set of two equations [8]:

$$\left.\begin{aligned} \mu P_1(\omega) + \gamma Q_1(\omega) + R_1(\omega) = 0 \\ \mu P_2(\omega) + \gamma Q_2(\omega) + R_2(\omega) = 0 \end{aligned}\right\} \quad (21)$$

By solving the set of equations (21) with respect to μ and γ the following results are achieved:

$$\mu = \frac{\Delta_1}{\Delta}, \quad \gamma = \frac{\Delta_2}{\Delta} \quad (22)$$

where

$$\left.\begin{aligned} \Delta &= \begin{bmatrix} P_1(\omega) & Q_1(\omega) \\ P_2(\omega) & Q_2(\omega) \end{bmatrix} \\ \Delta_1 &= \begin{bmatrix} -R_1(\omega) & Q_1(\omega) \\ -R_2(\omega) & Q_2(\omega) \end{bmatrix} \\ \Delta_2 &= \begin{bmatrix} P_1(\omega) & -R_1(\omega) \\ P_2(\omega) & -R_2(\omega) \end{bmatrix} \end{aligned}\right\} \quad (23)$$

It is obvious from equations (23) that Δ, Δ_1 and Δ_2 are odd functions. Therefore, considering equation (22), μ and γ are even functions. It follows that each of the parameters μ and γ has over-tracing values within the frequency region $-\infty \le \omega \le +\infty$, that is:

$$\left.\begin{aligned} \mu(+\omega) = \mu(-\omega) \\ \gamma(+\omega) = \gamma(-\omega) \end{aligned}\right\} \quad (24)$$

Then, if in the plain (μ, γ), the D-partitioning curve is plotted following the frequency increment from $-\infty$ to 0, the rest part of the curve, plotted for frequency increment from 0 to $+\infty$ is over-tracing the already plotted curve in reverse order (Figure 6). As stability region border is taken this part of this curve, that corresponds to the realistic parameter values.

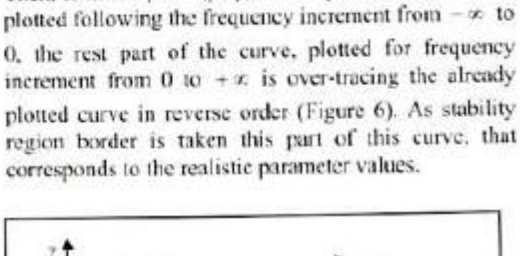

Figure 6. Overlapping curves from $\omega = -\infty$ to 0 and $\omega = 0$ to $+\infty$

The regions of the D-partitioning also depend on straight lines in the (μ, γ) plane, known as *special lines*. The special lines are plotted for two border frequencies $\omega = 0$ and $\omega = \infty$. Then the coefficients a_n and a_0 of the equation (1) depend directly on the parameters μ and γ and the equations of the special lines are obtained by:

$$a_n = 0, \qquad a_0 = 0 \quad (25)$$

The coefficient a_n determines a special line when $\omega = 0$, while the coefficient a_0 determines a special line when $\omega = \infty$.

Now, the D-partitioning regions could be determined by plotting the main D-partitioning curve, together with the special lines on the (μ, γ) plane. The locked regions between these parts of the curve, corresponding to realistic physically realized system parameters and the special lines are identified as the regions of stability. The realistic stable regions are also always located on the left-hand side of the D-partitioning curve, following the frequency increment.

4.2 Example of D-Partitioning by Two System Parameters

Suppose that again characteristic equation (8) is considered. Here, it is suggested that simultaneously two of the system's parameters are variable:

$$T_1 = \mu, \qquad K = \gamma \quad (26)$$

The objective is to determine the regions of variation of these two parameters, for which the system will be stable.

Equations (26) are substituted in (8), from where:

$$\left.\begin{aligned} \mu[T_2 T_3 s^3 + (T_2 + T_3)s^2 + s] + \\ + \gamma + T_2 T_3 s^2 + (T_2 + T_3)s + 1 = 0 \end{aligned}\right\} \quad (27)$$

By substituting $s = j\omega$ in equation (27) and taking into account (19), the set of equations (20) could be presented in the detailed form:

$$\left.\begin{aligned} P(j\omega) &= [T_2 T_3 (j\omega)^3 + (T_2 + T_3)(j\omega)^2 + j\omega] \\ Q(j\omega) &= 1 \\ R(j\omega) &= T_2 T_3 (j\omega)^2 + (T_2 + T_3) j\omega + 1 \end{aligned}\right\} \quad (28)$$

Considering equations (21), (22) and (23) the variable system's parameters can be determined as:

$$\left.\begin{aligned} \mu &= \frac{T_2 + T_3}{T_2 T_3 \omega^2 - 1} \\ \gamma &= \frac{(T_2 T_3 \omega^2 - 1)^2 + (T_2 + T_3)^2 \omega^2 + 1}{T_2 T_3 \omega^2 - 1} \end{aligned}\right\} \qquad (29)$$

Comparing the equation sets (22) and (29), it is obvious that:

$$\Delta = T_2 T_3 \omega^2 - 1 \qquad (30)$$

The determinant Δ becomes $\Delta = 0$ at a specific frequency $\omega = \omega_x$ that can be found out from equation (30):

$$\omega = \omega_x = \sqrt{\frac{1}{T_2 T_3}} \qquad (31)$$

It is clear from equations (22) that at $\omega = \omega_x$, when $\Delta = 0$, both system parameters are approaching infinity:

$$\mu(\omega_x) \to \infty, \quad \gamma(\omega_x) \to \infty. \qquad (32)$$

This implies that the main D-partitioning curve has an interruption, or a breakdown, at a frequency $\omega = \omega_x$. It consists of two parts, the first one is plotted within the frequency range $0 < \omega < \omega_x$, while the second one is obtained for $\omega_x < \omega < \infty$.

The special lines are determined by comparing the equations (1) and (27), identifying the coefficients a_n and a_o and equalizing them to zero:

$$\left.\begin{aligned} &a_n = \mu T_2 T_3 = 0, \qquad a_o = \gamma + 1 = 0 \\ &\text{or} \\ &\mu = 0 = Const. \qquad \gamma = -1 = Const. \end{aligned}\right\} \qquad (33)$$

The regions of stability are determined by the D-partitioning curve, defined by equations (29) and the special lines, defined by equations (33). For a better clarification and simplicity, first the functions $\mu(\omega)$ and $\gamma(\omega)$ are plotted, as shown in Figure 7(a) and Figure 7(b).

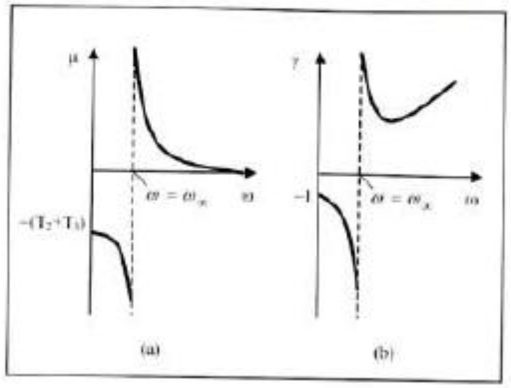

Figure 7. The graphical presentations of $\mu(\omega)$ and $\gamma(\omega)$ show the interruption (breakdown) of the curves at a frequency $\omega = \omega_x$.

Finally, by combining the $\mu(\omega)$ and $\gamma(\omega)$ curves and the special lines, the D-partitioning in the (μ, γ) plane is obtained.

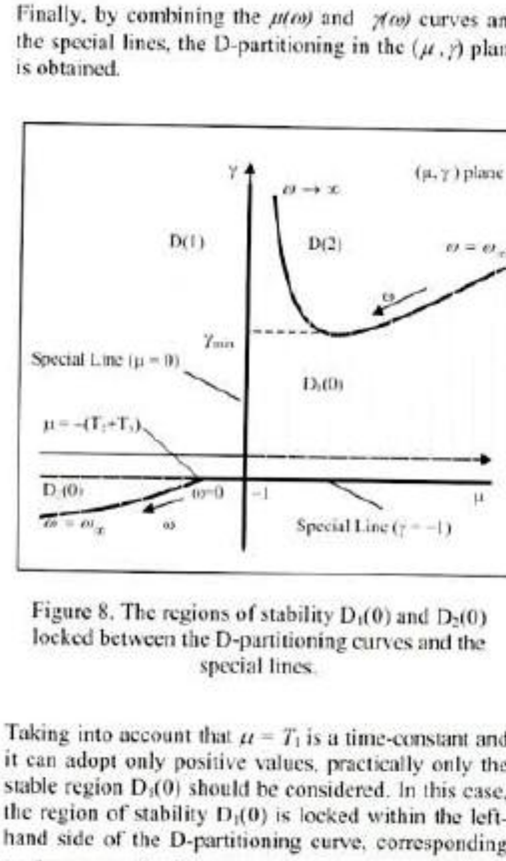

Figure 8. The regions of stability $D_1(0)$ and $D_2(0)$ locked between the D-partitioning curves and the special lines.

Taking into account that $\mu = T_1$ is a time-constant and it can adopt only positive values, practically only the stable region $D_1(0)$ should be considered. In this case, the region of stability $D_1(0)$ is locked within the left-hand side of the D-partitioning curve, corresponding to frequency rise from $\omega = \omega_x$ to $\omega \to \infty$ and the special line $\gamma = -1$. Further, the conclusion is that for small values of the gain $K < \gamma_{min}$, the system is stable for any values of the time-constant $\mu = T_1$. The value of γ_{min} can be easily determined by taking the first derivative of the second equation of the set (29). Large gain values $K > \gamma_{min}$ could be employed only for very small or very large values of the time-constant $\mu = T_1$.

5 CONCLUSIONS

The method of the D-partitioning is essential for controller design and analysis in robust control systems and control systems with real parametric uncertainty [12,13,14]. Here, one of the key features is the concept of robustness. Instead of a nominal system, we study a family of systems and we say that a certain property (e.g., stability) is robustly satisfied if it is satisfied for all members of the family [15,16,17]. Based on the theory of root positioning of the characteristic equation, the method of the D-partitioning determines the limits of variation of the parameters within which the system remains stable. The regions of stability are obtained by plotting the D-partitioning curves and in some cases special lines. The method can be considered as another opportunity for determination of stability of systems with variable parameters. The contribution of this paper is, by adopting some of the initial theoretical ideas, to develop further the mathematical analysis and to show the practical procedures for implementation of the method. When the method of the D-partitioning is well clarified and its application simplified, it has the advantages in terms of a clear and graphical display of all regions of parameter variations for which the system remains stable.

Laboratory experiments have been performed, by simulating the operation of a third order control system. The system used, being a hardware model, gives the opportunity of variation of any of its parameters and has a characteristic equation of the type described by equation (8). First, the system of the laboratory model has been examined for stability, if one of its parameters is variable. The test was performed for variation of the system's gain and then for one of the system's time constants. Further, the system was tested by varying simultaneously two of its parameters (gain and time constant). The outcomes from the experiments match completely the theoretically obtained results. In this way the validity of the application of the D-partitioning method has been verified and proven for a number of cases of a third order system with variable parameters. By drawing a general conclusion from the theoretical and laboratory results, the method of the D-partitioning can be implemented practically for any n-order system with one or two variable parameters.

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